

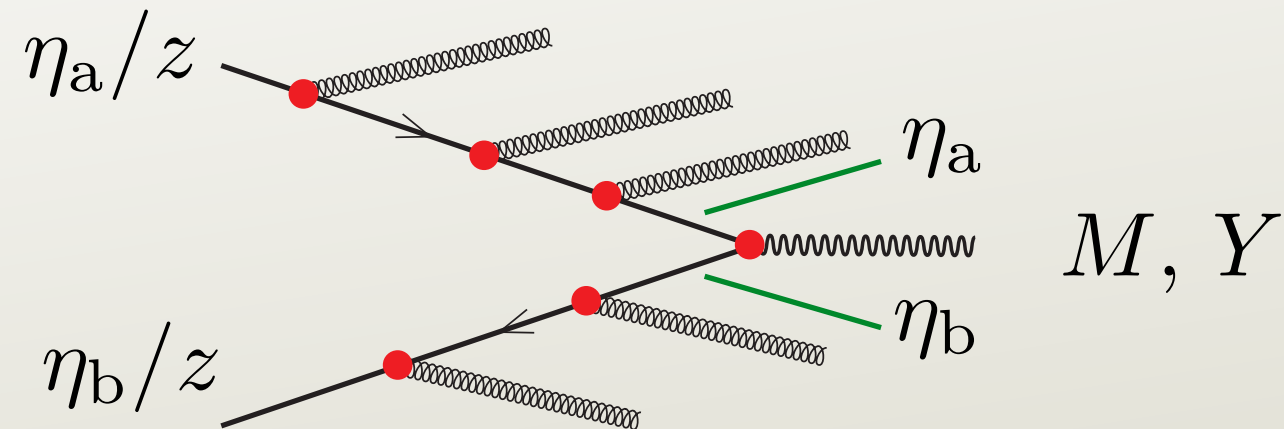
Threshold logarithms in a parton shower

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DESY

work with Dave Soper, University of Oregon

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- Consider the Drell-Yan process with dimuon rapidity Y and mass M .



- There are logarithms of $(1 - z)$:

$$\int_0^1 dz \, f_{a/A}(\eta_a/z, \mu_F^2) \left\{ \delta(1 - z) + C\alpha_s \left[\frac{\log(1 - z)}{1 - z} \right]_+ + \dots \right\}$$

- There is a large literature on summing these logarithms starting with Sterman (1987).

DEDUCTOR

- <http://pages.uoregon.edu/soper/deductor/>
- Dipole shower.
- In principle, uses quantum density matrix in color & spin.
- LC+ approximation for color.
- Z. Nagy and D. E. Soper,
“Summing threshold logs in a parton shower,”
arXiv:1605.05845 [hep-ph]

Coming in DEDUCTOR

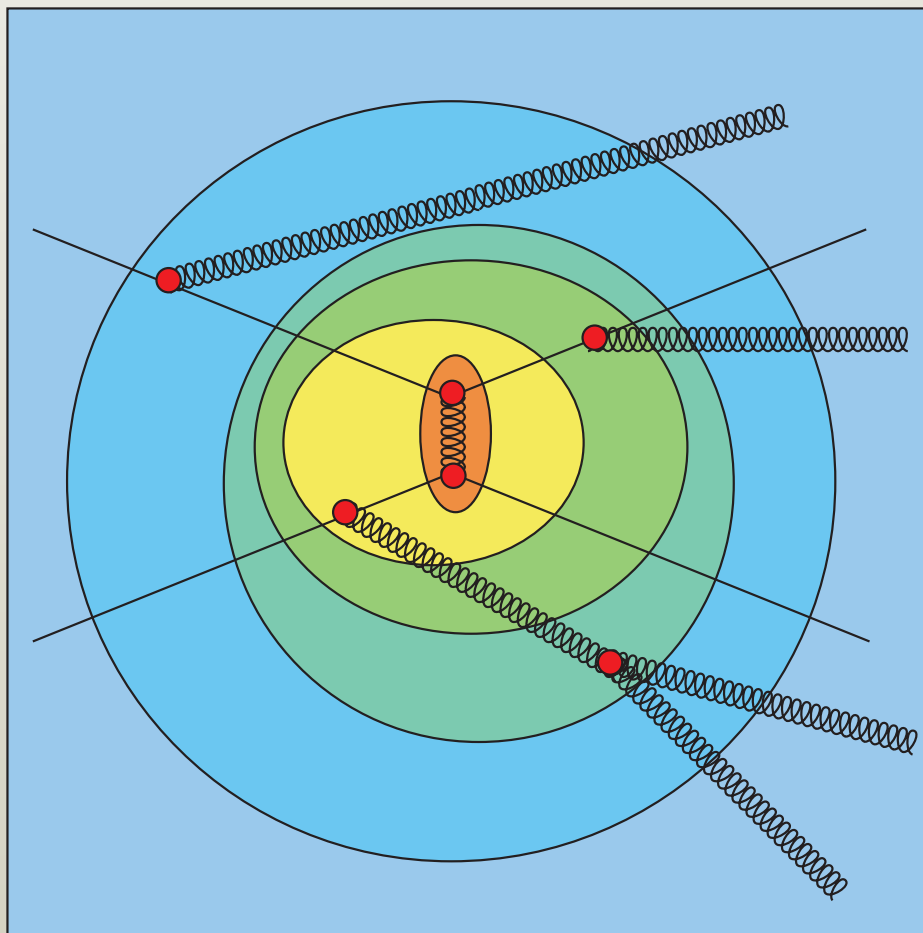
- Perturbative improvement to LC+ approximation.
- Quantum spin.
- Interface to hadronization model.

Now in DEDUCTOR

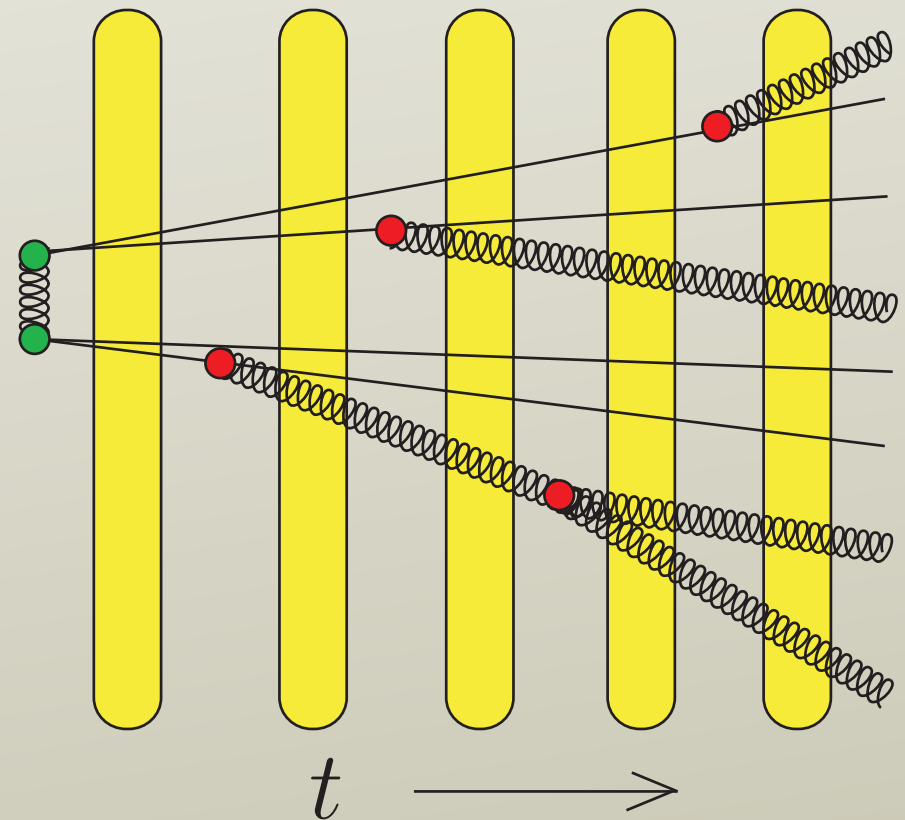
- Threshold logs (this talk).
- Choices, e.g. for definition of ordering variable.

Shower evolution

- Showers develop in “shower time.”
- Hardest interactions first.



Real time picture



Shower time picture

Shower ordering variable

- Originally, PYTHIA used virtuality to order splittings.
- Now, PYTHIA and SHERPA use “ k_T .”
- DEDUCTOR uses Λ ,

$$\Lambda^2 = \frac{(\hat{p}_l + \hat{p}_{m+1})^2}{2p_l \cdot Q_0} Q_0^2 \quad \text{final state,}$$

$$\Lambda^2 = - \frac{(\hat{p}_a - \hat{p}_{m+1})^2}{2p_a \cdot Q_0} Q_0^2 \quad \text{initial state}$$

- Here Q_0 is a fixed timelike vector.

Contrast with SCET

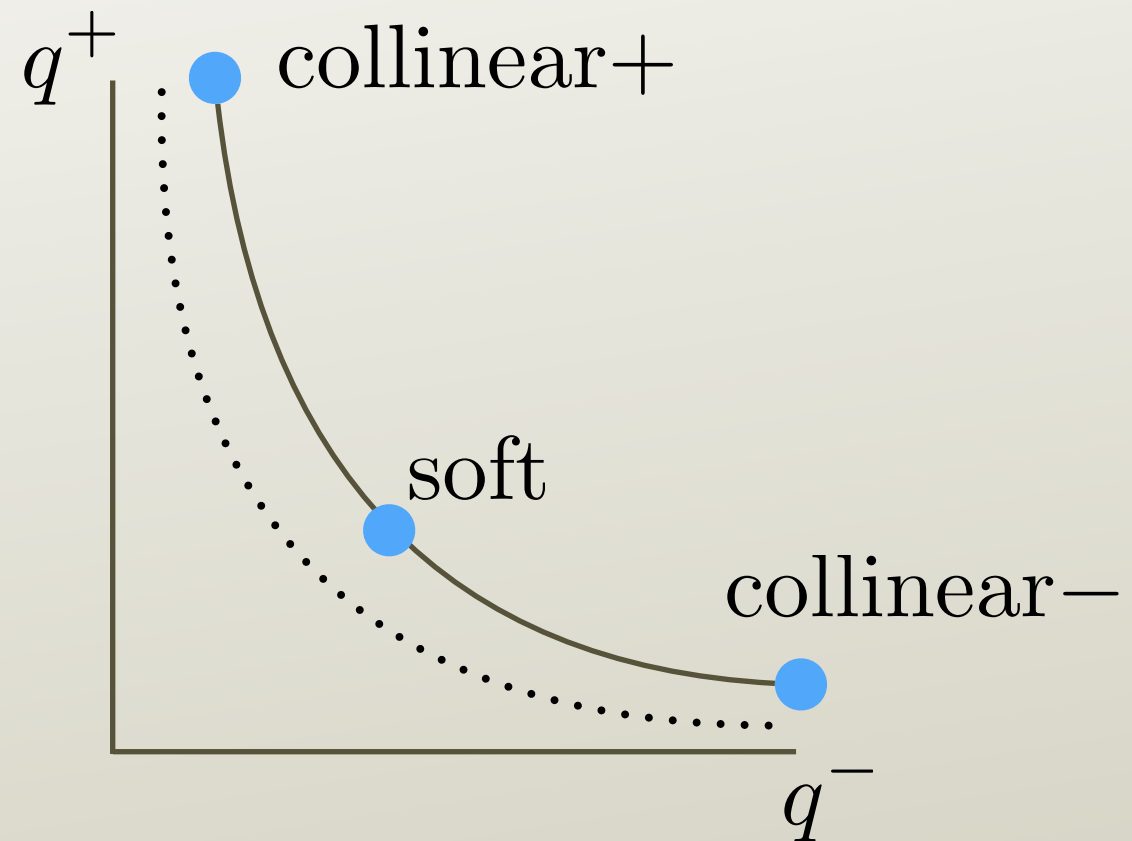
- SCET distinguishes multiple regions.

$$(q^+, q^-, q_\perp) = (1, \lambda^2, \lambda)Q \quad \text{collinear+}$$

$$(q^+, q^-, q_\perp) = (\lambda^2, 1, \lambda)Q \quad \text{collinear-}$$

$$(q^+, q^-, q_\perp) = (\lambda, \lambda, \lambda)Q \quad \text{soft}$$

$$\lambda \ll 1$$



- A parton shower has just larger λ and smaller λ .
- Shower evolution from

$$\frac{d}{d\lambda}$$

The shower state

- Here I am ignoring spin.
- To describe the state at shower time t based on an ensemble of runs of the program, use the density operator in color space

$$\rho(\{p, f\}_m, t) = \sum_{\{c\}_m, \{c'\}_m} \rho(\{p, f, c', c\}_m, t) |\{c\}_m\rangle\langle\{c'\}_m|$$

- Here $\rho(\{p, f, c', c\}_m, t)$ is the probability to find the system with momenta and flavors $\{p, f\}_m$ in this color state.
- Denote this function by $|\rho(t)\rangle$.

Evolution equation

$$|\rho(t)\rangle = \mathcal{U}_S(t, t_0) |\rho(t_0)\rangle$$

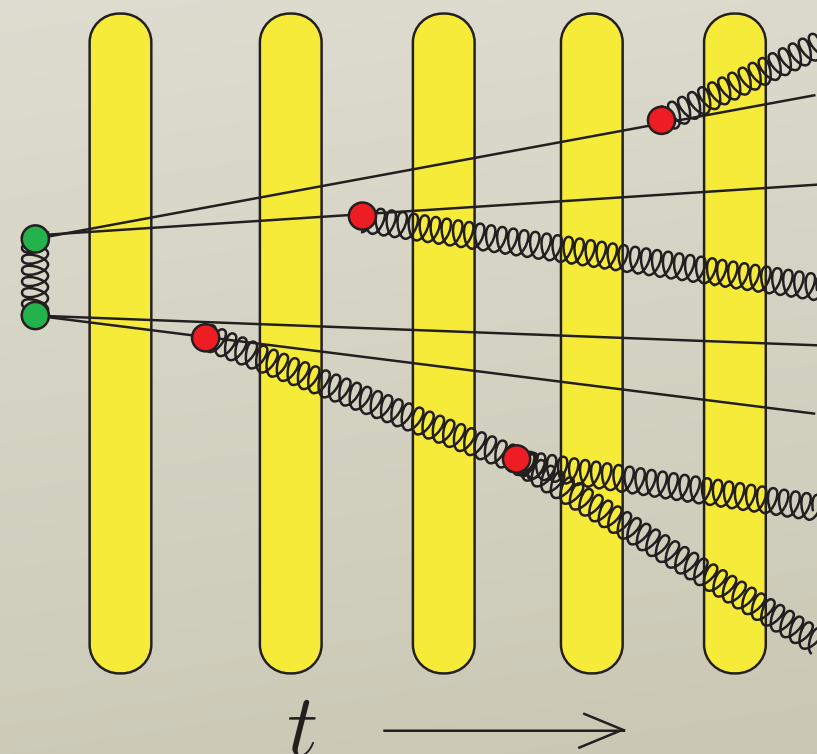
$\mathcal{H}_I(t)$ = splitting operator

$$\frac{d}{dt} \mathcal{U}_S(t, t') = [\mathcal{H}_I(t) - \mathcal{S}(t)] \mathcal{U}_S(t, t') \quad \mathcal{S}(t) = \text{no-splitting operator}$$

$$\mathcal{U}_S(t, t') = \mathcal{N}_S(t, t') + \int_{t'}^t d\tau \mathcal{U}_S(t, \tau) \mathcal{H}_I(\tau) \mathcal{N}_S(\tau, t')$$

where

$$\mathcal{N}_S(\tau, t') = \mathbb{T} \exp \left[- \int_{t'}^{\tau} d\tau' \mathcal{S}(\tau') \right]$$



Role of parton distributions

- $\rho(t)$ contains a factor of parton distributions,

$$|\rho(t)\rangle = \mathcal{F}(t)|\rho_{\text{pert}}(t)\rangle$$

- Here $|\rho_{\text{pert}}(t)\rangle$ is $|M\rangle\langle M|$ from Feynman diagrams.
- We include parton distribution functions at the current scale.

$$\begin{aligned} \mathcal{F}(t)|\{p, f, c', c\}_m\rangle \\ = \frac{f_{a/A}(\eta_a, \mu_a^2(t)) f_{b/B}(\eta_b, \mu_b^2(t))}{4n_c(a)n_c(b) 4\eta_a\eta_b p_B \cdot p_B} |\{p, f, c', c\}_m\rangle \end{aligned}$$

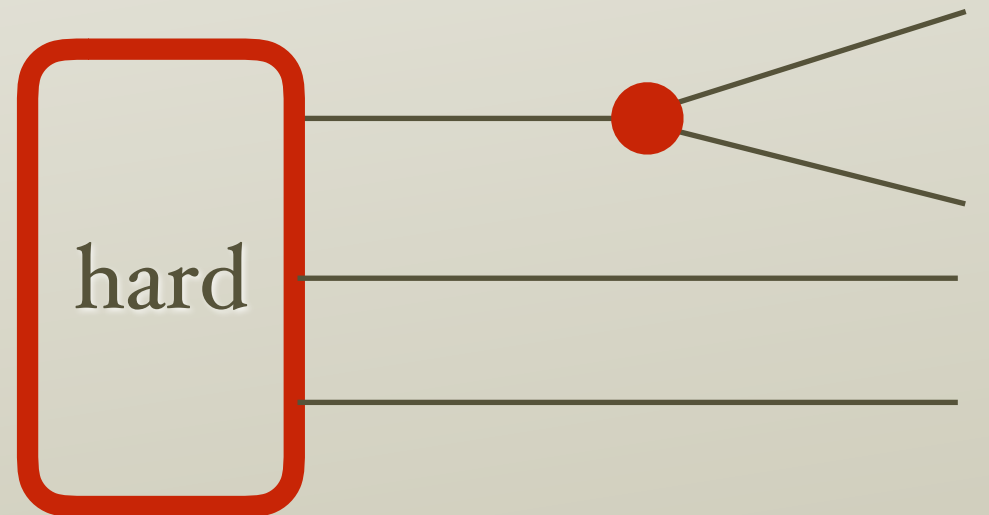
Evolution for the perturbative state

$$|\rho(t)\rangle = \mathcal{F}(t)|\rho_{\text{pert}}(t)\rangle$$

$$\frac{d}{dt}|\rho_{\text{pert}}(t)\rangle = [\mathcal{H}_I^{\text{pert}}(t) - \mathcal{S}^{\text{pert}}(t)]|\rho_{\text{pert}}(t)\rangle$$

- Calculate $\mathcal{H}_I^{\text{pert}}(t)$ from Feynman diagrams.
- Then use

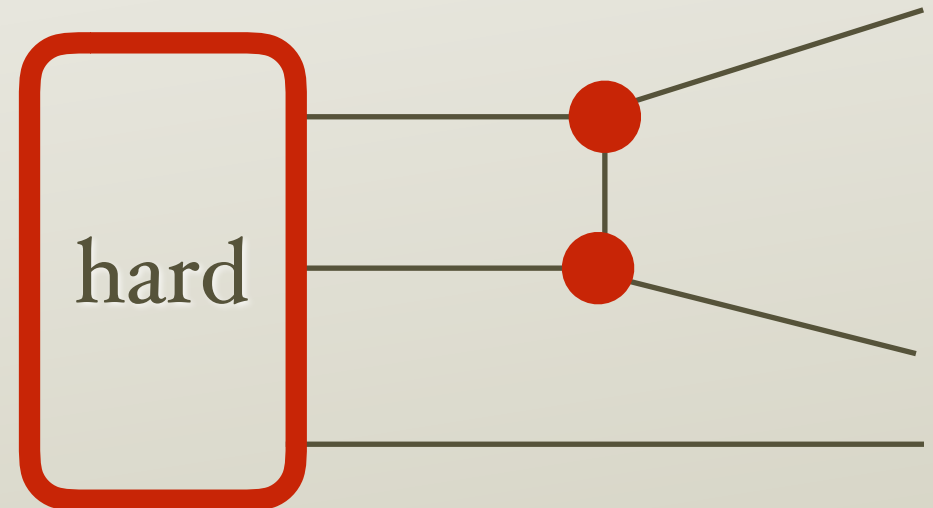
$$\mathcal{H}_I(t) = \mathcal{F}(t)\mathcal{H}_I^{\text{pert}}(t)\mathcal{F}(t)^{-1}$$



$$|\rho(t)\rangle = \mathcal{F}(t)|\rho_{\text{pert}}(t)\rangle$$

$$\frac{d}{dt}|\rho_{\text{pert}}(t)\rangle = [\mathcal{H}_I^{\text{pert}}(t) - \mathcal{S}^{\text{pert}}(t)]|\rho_{\text{pert}}(t)\rangle$$

- Calculate $\mathcal{S}^{\text{pert}}$ from Feynman diagrams.
- Then use



$$\mathcal{S}(t) = \mathcal{S}^{\text{pert}}(t) - \mathcal{F}(t)^{-1} \left[\frac{d}{dt} \mathcal{F}(t) \right]$$

Including threshold logs

- A parton shower can sum logarithms if you let it.
- I can show you the main idea.

What not to do

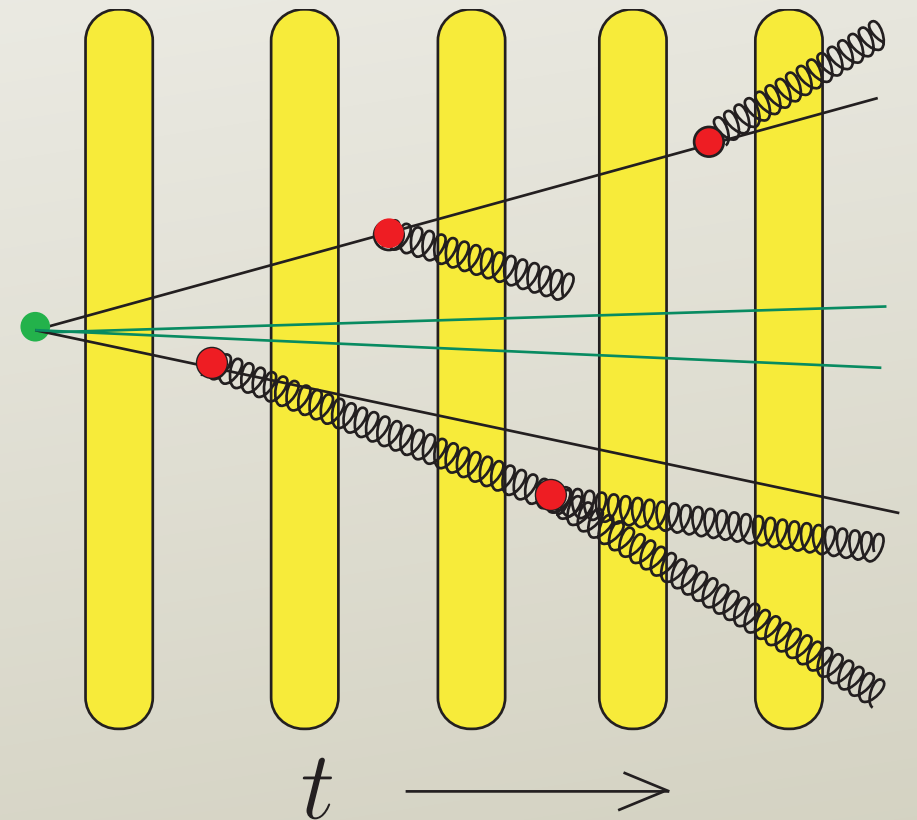
- Suppose that the shower state evolves according to

$$|\rho(t)\rangle = \mathcal{U}_{\mathcal{V}}(t, t') |\rho(t')\rangle$$

$$\frac{d}{dt} \mathcal{U}_{\mathcal{V}}(t, t') = [\mathcal{H}_I(t) - \mathcal{V}(t)] \mathcal{U}_{\mathcal{V}}(t, t')$$

$$\mathcal{H}_I(t) = \text{splitting operator}$$

$$\mathcal{V}(t) = \text{no-splitting operator}$$



- We calculate $\mathcal{V}(t)$ from $\mathcal{H}_I(t)$ so that the inclusive cross section does not change during the shower.

What to do

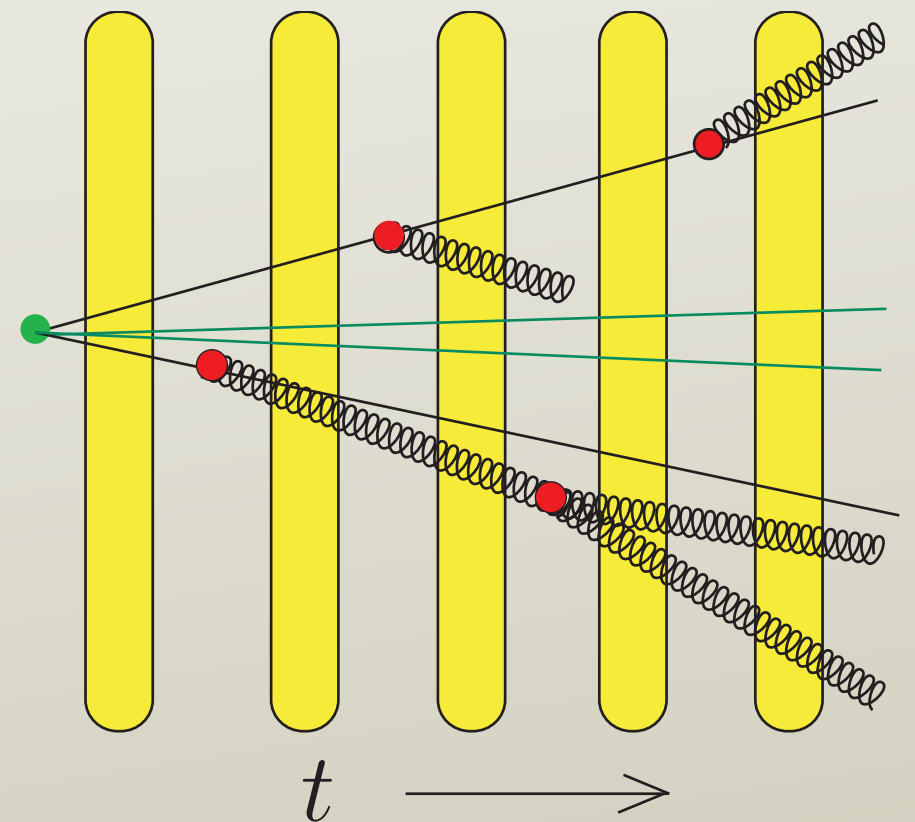
- The shower state evolves in shower time.

$$|\rho(t)\rangle = \mathcal{U}_S(t, t') |\rho(t')\rangle$$

$$\frac{d}{dt} \mathcal{U}_S(t, t') = [\mathcal{H}_I(t) - \mathcal{S}(t)] \mathcal{U}_S(t, t')$$

$\mathcal{H}_I(t)$ = splitting operator

$\mathcal{S}(t)$ = virtual splitting
and parton evolution



- We simply calculate $\mathcal{S}(t)$ from one loop virtual graphs plus parton evolution.

What happens

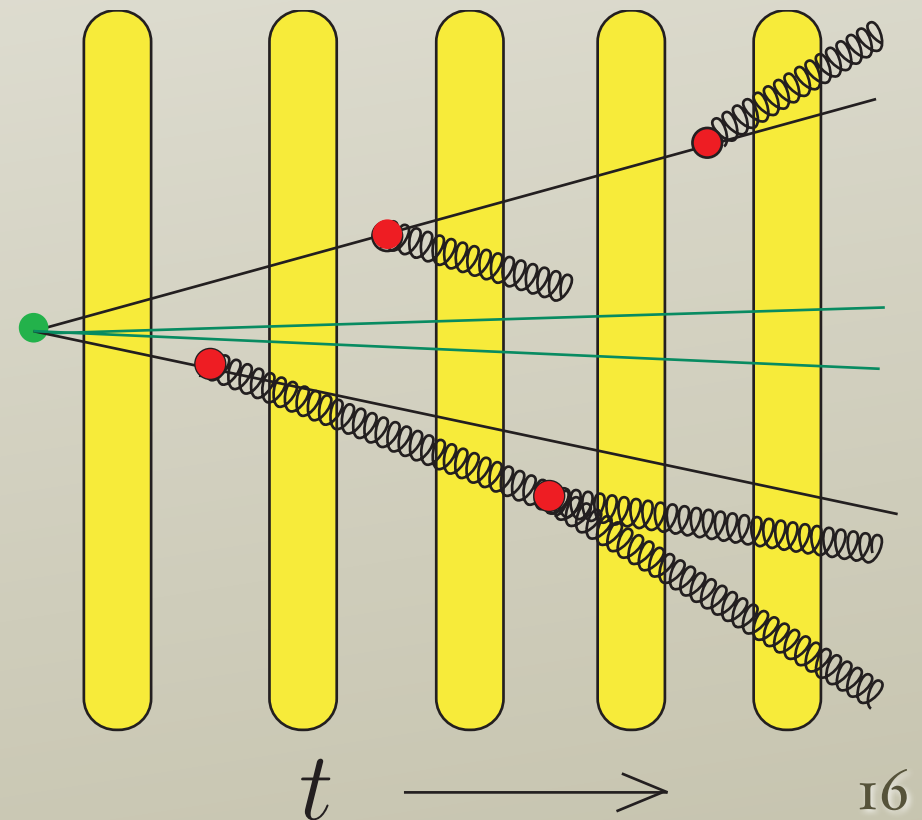
$$\mathcal{U}_S(t, t_0) = \mathcal{N}_S(t, t_0) + \int_{t_0}^t d\tau \mathcal{U}_S(t, \tau) \mathcal{H}_I(\tau) \mathcal{N}_S(\tau, t_0)$$

$$\mathcal{N}_S(t_2, t_1) = \mathbb{T} \exp \left[\int_{t_1}^{t_2} d\tau \left[-\mathcal{V}(\tau) + (\mathcal{V}(\tau) - \mathcal{S}(\tau)) \right] \right]$$

- Within the LC+ approximation, the operators commute.
- There is an extra factor

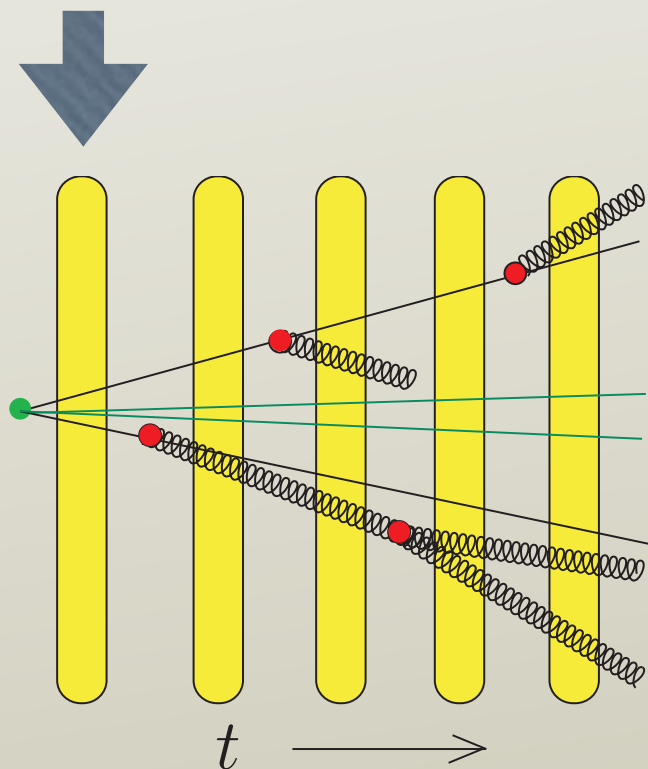
$$\exp \left[\int_{t_1}^{t_2} d\tau (\mathcal{V}(\tau) - \mathcal{S}(\tau)) \right]$$

that changes the cross section.



The most important term

- Look at the Drell-Yan process.
- Look at the factor for line “a” just after the hard interaction.
- Assume that no real gluons have been emitted yet.



- Use y = dimensionless virtuality variable (with $y \ll 1$) and z = momentum fraction.

- Result: almost everything cancels.
- Two terms do not quite cancel.

$$\begin{aligned}
& [\mathcal{V}_a(t) - \mathcal{S}_a(t)] | \{p, f, c', c\}_m) = \\
& \left\{ \int_0^{1/(1+y)} \frac{dz}{z} \sum_{\hat{a}} \frac{\alpha_s}{2\pi} \left(\frac{f_{\hat{a}/A}(\eta_a/z, Q^2 y)}{f_{a/A}(\eta_a, Q^2 y)} P_{a\hat{a}}(z) - \delta_{a\hat{a}} \frac{2C_a z}{1-z} \right) [1 \otimes 1] \right. \\
& \quad - \int_0^1 \frac{dz}{z} \sum_{\hat{a}} \frac{\alpha_s}{2\pi} \left(\frac{f_{\hat{a}/A}(\eta_a/z, Q^2 y)}{f_{a/A}(\eta_a, Q^2 y)} P_{a\hat{a}}(z) - \delta_{a\hat{a}} \frac{2C_a z}{1-z} \right) [1 \otimes 1] \\
& \quad \left. + \dots \right\} | \{p, f, c', c\}_m)
\end{aligned}$$

- $z < 1/(1+y)$ comes from splitting kinematics.
- $z < 1$ comes from parton evolution.

- This gives

$$[\mathcal{V}_a(t) - \mathcal{S}_a(t)]|\{p, f, c', c\}_m) =$$

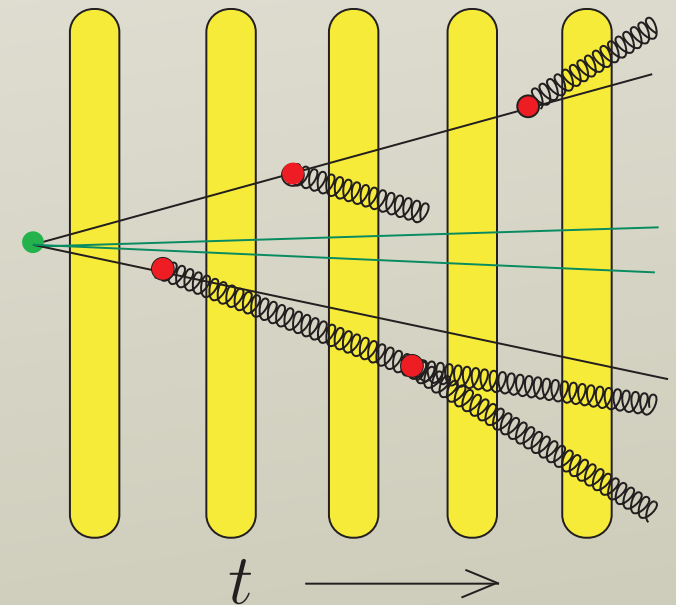
$$\left\{ \int_{1/(1+y)}^1 dz \frac{\alpha_s}{2\pi} \frac{2C_a}{1-z} \left(1 - \frac{f_{a/A}(\eta_a/z, Q^2 y)}{f_{a/A}(\eta_a, Q^2 y)} \right) [1 \otimes 1] \right.$$

$$\left. + \cdots \right\} |\{p, f, c', c\}_m)$$

- The $1/(1-z)$ factor creates the “threshold log.”
- But the parton factor contains a factor $(1-z)$ so there is no actual log.
- For $y \ll 1$, this contribution is suppressed by a factor of y .
- But, the parton factor can be large, so we keep this.

Conclusion on threshold logs

- We find simple and intuitive leading order formulas.
- This is in the context of a leading order parton shower not “NLO,” “NLL” or “NNLL.”
- This is implemented as part of DEDUCTOR.
- The summation applies to all hard processes.
- The shower sums the threshold logs jointly with other large logs.

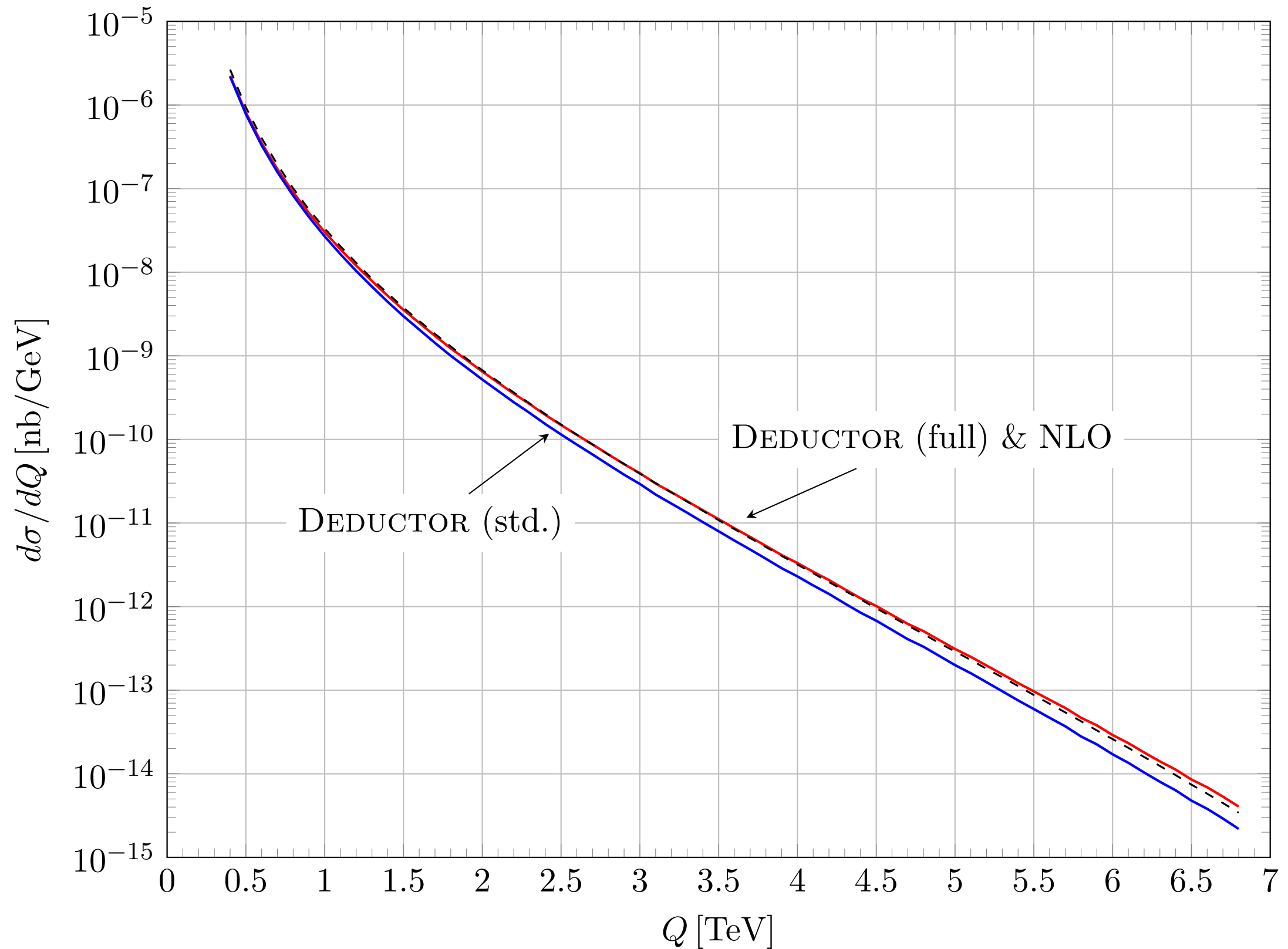


Some details

- There are more terms, some with non-trivial color.
- We need to account for switch to parton distributions based on virtuality instead of transverse momentum as the measure of hardness.

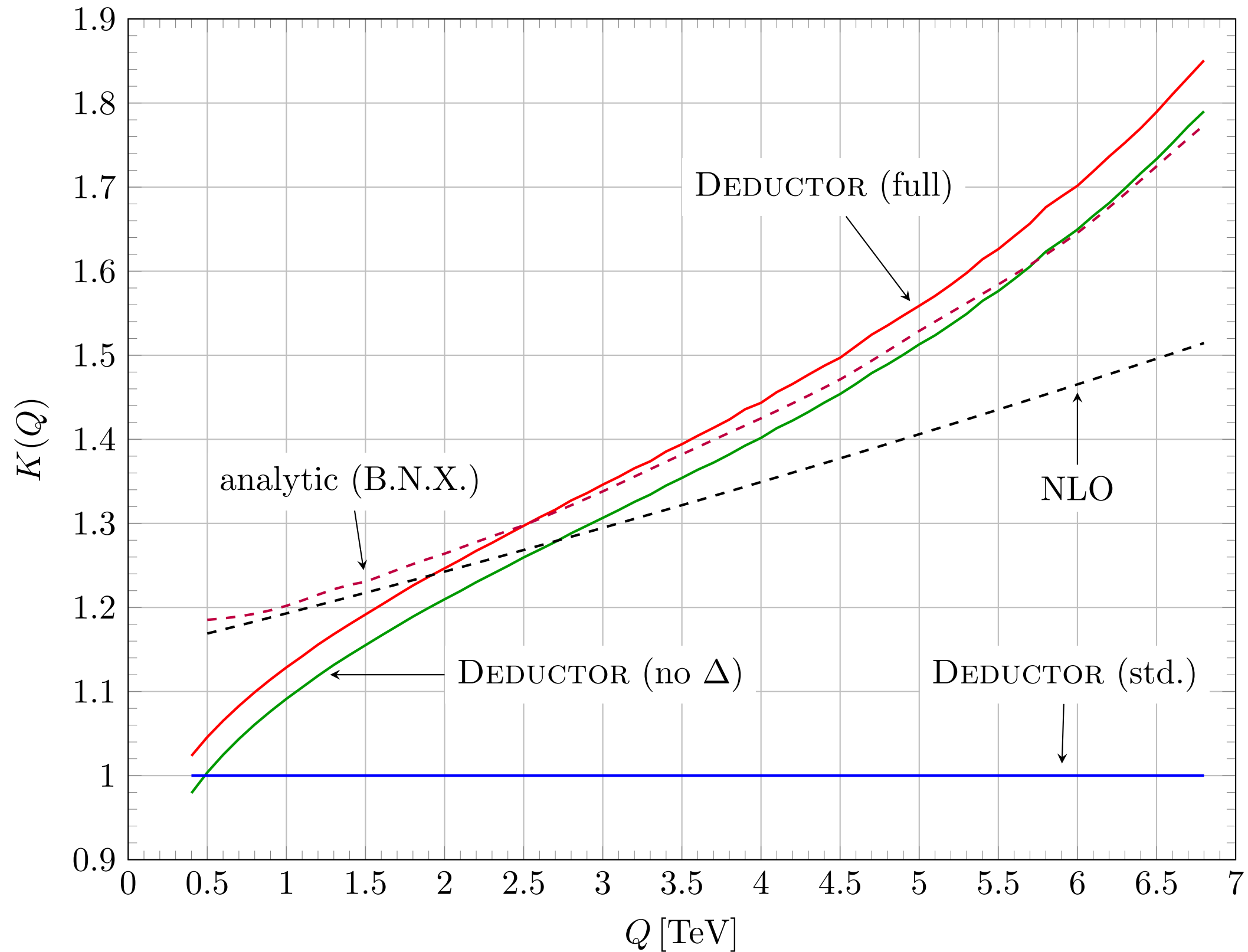
Drell-Yan

Drell-Yan cross section



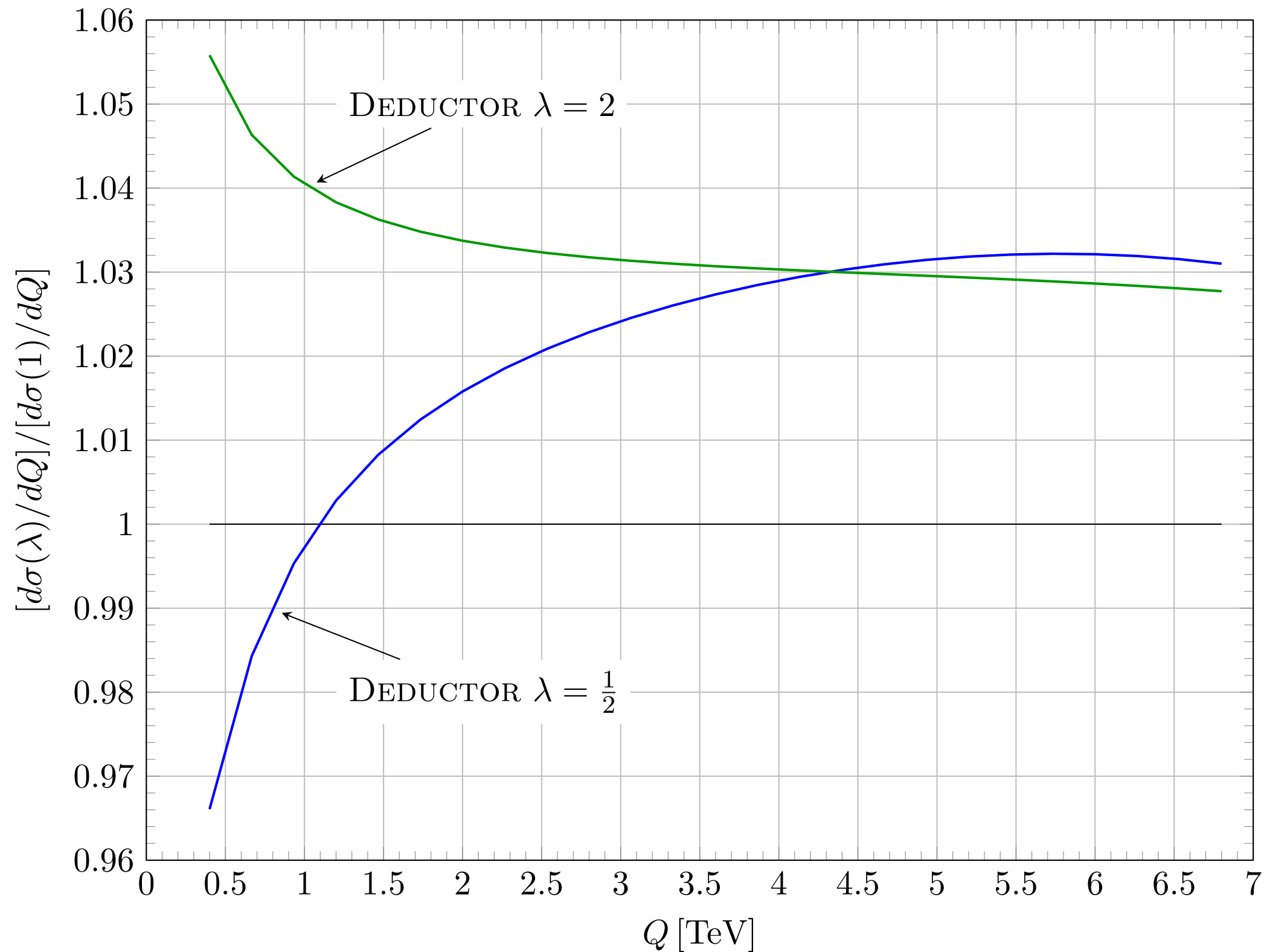
Drell-Yan

Ratios of Drell-Yan cross sections



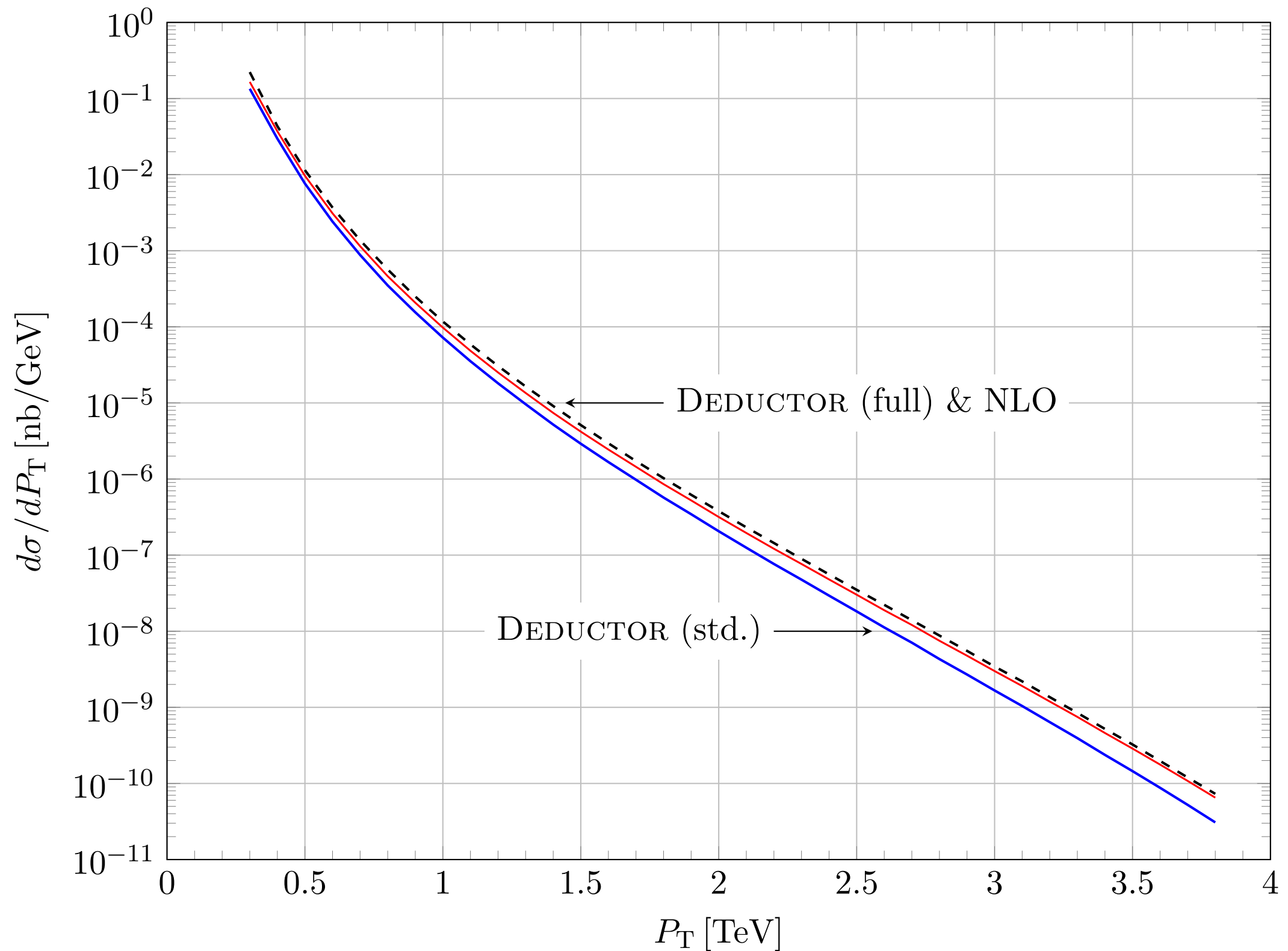
Drell-Yan

Scale dependence of Drell-Yan cross section



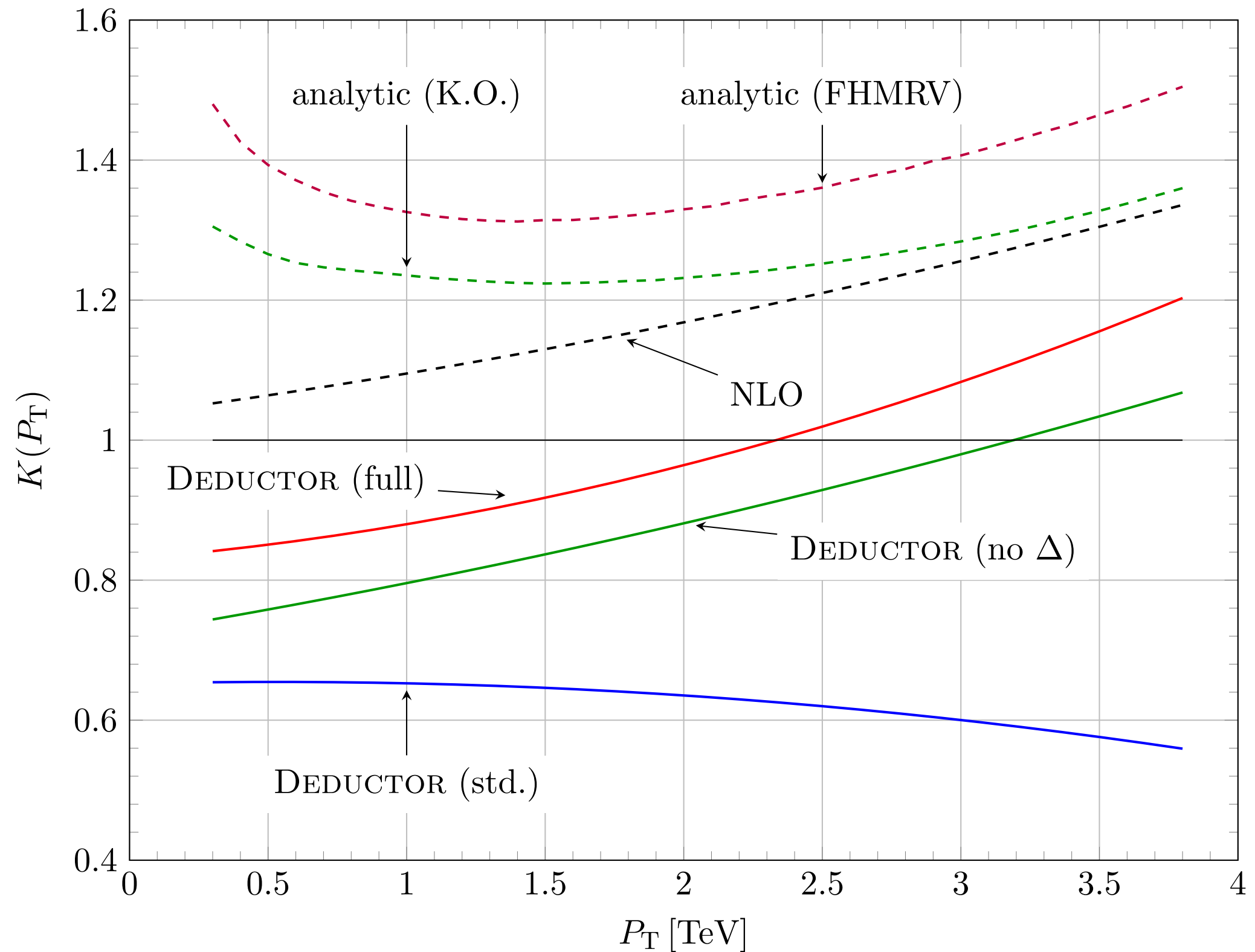
Jet Production

One jet inclusive cross section



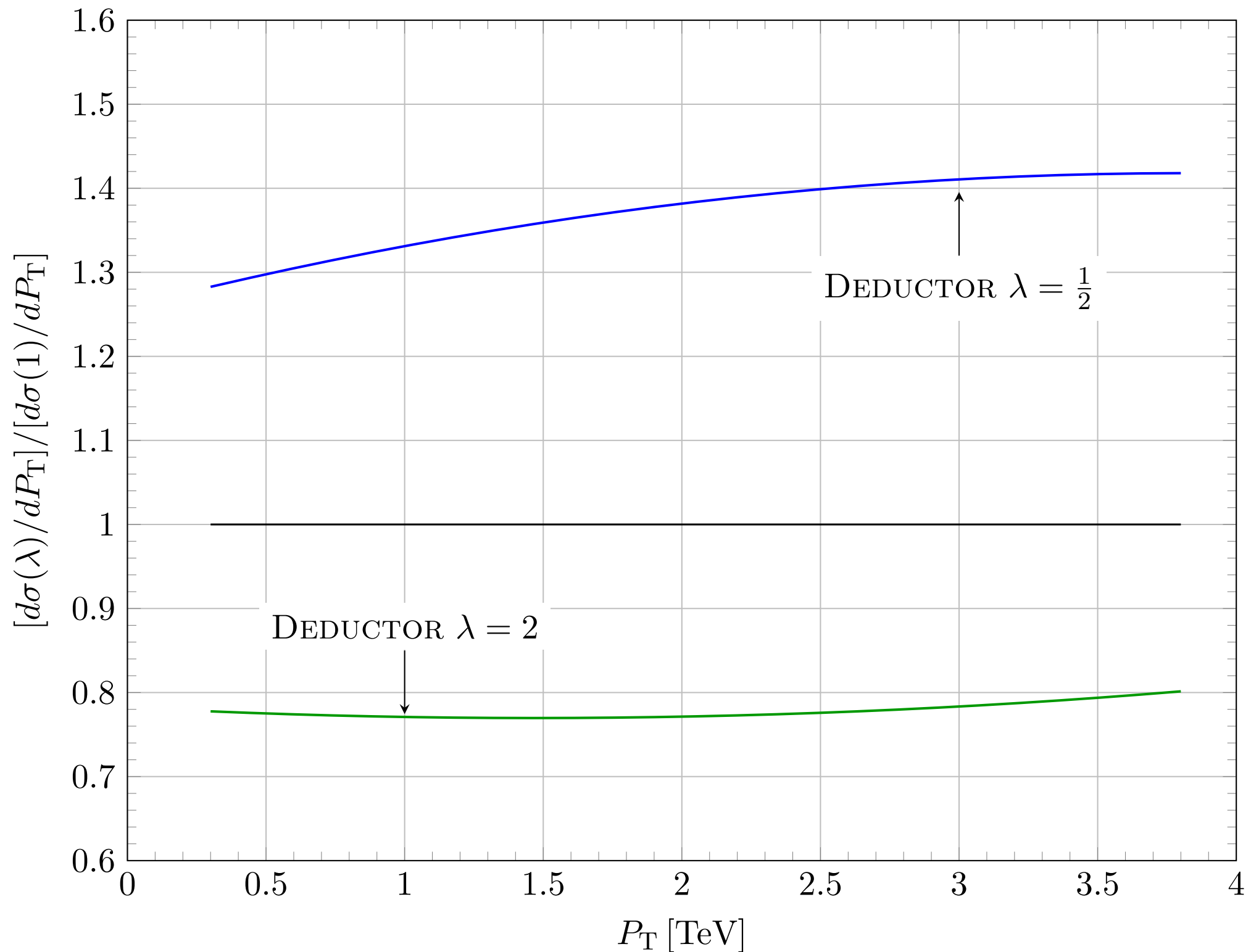
Jet Production

Ratios of one jet inclusive cross section



Jet Production

Scale dependence of one jet inclusive cross section



General conclusion

- Parton shower event generators can sum logarithms.
- They are leading order, so not as precise as SCET.
- But they are useful because they are more general.
- Summing threshold logs with a parton shower is possible.